[12.4] Problem Statement: Let  be a scalar field on an *n‑*manifold **M**. Show that the (*n*–1) -dimensional plane elements determined by d are tangential to the family of (*n*–1)-dimensional surfaces of constant Φ.

There were no posted solutions for this problem so I’ll make an attempt.

**Solution:**

It suffices to show this for every point of the manifold.

Let P be a point of **M**.

Let XP be the (*n*–1)-dimensional plane element at P determined by d.

Let ** be a vector field.

By Penrose’s definition of the (*n*–1)-dimensional plane element at P determined by d, the direction of ** belongs to XP iff . But , the rate of change of  in the direction of **.

This implies that Φ is constant on XP

Thus XP is tangential to the family of (n–1)-dimensional surfaces of constant Φ.✔