[12.4] Problem Statement: Let ****** be a vector field, ** a scalar field, and d** the covector with components . (i) At each point of **M**, a (non-zero) covector ****** determines an (n-1) dimensional plane element. (ii) When , these (n-1) plane elements are tangential to the family of (n-1) dimensional surfaces of constant *Φ*.

Solution:

1. In general,  and



First, consider  = dx1.

 Thus **(**) = d*x*1 represents the magnitude of the rate of change of ** in the *x*1 direction. That is, at a point P∈ **M** it represents the magnitude of the rate of change of ** at P when *x*2, *x*3, …, *x*n are held constant. Thus [**(**)] (**) represents the (n‑1)‑dimensional plane element <*x*2, *x*3, …, *x*n> at P.

Now consider  in general. **(**) represents the magnitude of the rate of change of **in the direction of **, namely where {*xk*} are unit vectors along the *xk*-axes. That is, like *dx*1(**), [()] () represents a vector at P, and thus it determines the (n‑1)‑dimensional plane element at P perpendicular to the vector.

1. Let ** = *d*. We know  So when  ** points along a direction of constant **. Thus, the (n-1)-dimensional plane element at P generated by ** in part (i) is a hyperplane where ** is constant.